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COMPUTER-SYNTHESIZED OPTICAL ELEMENTS FOR CORRECTING OPTICAL SYSTEMS

COMPUTER-SYNTHESIZED OPTICAL ELEMENTS FOR CORRECTING ABERRATIONS OF IMAGING SYSTEMS

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Abstract—An analysis is presented of the use of planar computer-optical elements to correct the aberrations of single and multicomponent lens systems. In the paraxial approximation, an equation for the phase function of a corrector plate is derived. For a correcting device synthesized with a finite number of discretization points and phase quantization levels, estimates of the Strehl definition, resolution and mean square aberration are obtained with account of the diffraction phenomena. Qualitative tables illustrate the effectiveness of aberration control achieved with computer-synthesized optical elements for a thin lens.

A substantial improvement of wave aberration control offered by systems with phase layers [1, 2] has been hard to achieve because of difficulties in the deposition of phase layers into zones with a desired profile. This shortcoming may be overcome with the aid of "computer optics" technology developed for the synthesis of wavefront correctors [3]. For axially symmetric holographic optical elements, the fabrication procedures have been discussed by Gan [4] and Bobrov *et al.* [5].

The present paper considers the effect of discretization and phase quantization on the image quality by correcting devices which in general have no rotational symmetry. The analysis is illustrated by estimates of the Strehl definition and the resolution of a corrector plate designed to cancel the aberrations of a single lens. For the sake of simplicity the calculation is carried out for the case of an infinitely distant object.

(i) Let a thin optical element placed in a domain G of the plane $\mathbf{u} = (u, v)$ form an image in the region D of the plane $\mathbf{x} = (x, y)$ at a distance f_0. The object is described by the angular coordinates $\theta = (\theta_x, \theta_y)$,

$$\theta_x = \sin\theta_1, \qquad \theta_y = \sin\theta_2, \tag{1}$$

where $\pi/2 - \theta_1$ and $\pi/2 - \theta_2$ are the angles that a ray makes with the u and v axes, respectively. Denote by $b(\mathbf{u}, \boldsymbol{\theta})$ the eiconal transmission function of a thin element, i.e. the variation of optical path length for the ray passing point $\mathbf{u}$ at an angle θ.

In the Gaussian approximation, an object point $\boldsymbol{\theta}$ is imaged to the point

$$\mathbf{x}_\theta = R_\theta \theta, \tag{2}$$

where $R_\theta = f_0\sqrt{1 - \boldsymbol{\theta}^2}$ is the radius of the Gaussian reference sphere. (3)

By virtue of the eiconal equation [6] one can readily obtain for the transverse spherical aberration vector

$$\boldsymbol{\kappa}(\mathbf{u}, \boldsymbol{\theta}) = \mathbf{u} - \mathbf{x}_0 + f_0 \frac{\boldsymbol{\theta} + \nabla_u b(\mathbf{u}, \boldsymbol{\theta})}{\sqrt{1 - [\boldsymbol{\theta} + \nabla_u b(\mathbf{u}, \boldsymbol{\theta})]^2}}, \tag{4}$$

where $\nabla_u = \left(\dfrac{\partial}{\partial u}, \dfrac{\partial}{\partial v}\right)$.

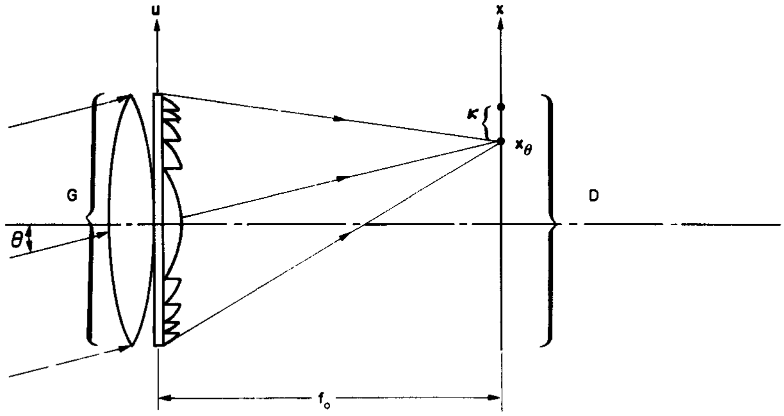

Fig. 1.

The wave aberration can be recovered [6, 7] by the formula

$$B(\mathbf{u}, \boldsymbol{\theta}) = B_0 + \frac{1}{R_\theta} \int_{\mathbf{u}_0}^{\mathbf{u}} \kappa(\mathbf{u}', \boldsymbol{\theta})\, d\mathbf{u}', \tag{5}$$

where the integral is taken along any curve passing through points $\mathbf{u}_0$ and $\mathbf{u}$.

In the paraxial approximation, when $\boldsymbol{\theta}$ tends to zero and $|\nabla_u b| \ll 1$, we have

$$\boldsymbol{\kappa}(\mathbf{u}, \boldsymbol{\theta}) \simeq \mathbf{u} + f_0 \nabla_u b(\mathbf{u}, \boldsymbol{\theta}), \tag{6}$$

that is,

$$B(\mathbf{u}, \boldsymbol{\theta}) = B_0 + \frac{\mathbf{u}^2}{2f_0} + b(\mathbf{u}, \boldsymbol{\theta}). \tag{7}$$

Therefore, the paraxial wave aberration of a thin element coincides with the eiconal transmission function $b(\mathbf{u}, \boldsymbol{\theta})$ of this element less the transmission function $-\mathbf{u}^2/2f_0$ of the paraxial lens.

(ii) Let the thin optical element consist of a thin lens of focal length f_0 (a singlet or a cemented piece) tightly joined in tandem with a correcting device to be synthesized, as shown in Fig. 1. If $b_l(\mathbf{u}, \boldsymbol{\theta})$ is the wave aberration of the lens, then the wave aberration of the corrector plate $B_c(\mathbf{u}, \boldsymbol{\theta})$ is described by the equation

$$B_c(\mathbf{u}, \boldsymbol{\theta}) \simeq -b_l(\mathbf{u}, \boldsymbol{\theta}), \tag{8}$$

where the equality is achieved at some $\boldsymbol{\theta}$ for all $\mathbf{u}$.

For axially symmetric optical systems of free aperture $2a$, the function $B_c(\mathbf{u}, \boldsymbol{\theta})$ may be written, accurate to third-order aberrations, as [6]

$$B_c(\mathbf{u}, \boldsymbol{\theta}) = b_{c0} + B\frac{1}{4}\left(\frac{\mathbf{u}^2}{a^2}\right)^2 + C\left(\frac{\boldsymbol{\theta}\cdot\mathbf{u}}{a\theta_m}\right)^2 + \frac{1}{2}D\frac{\boldsymbol{\theta}^2\mathbf{u}^2}{\theta_m^2 a^2} - E\frac{\boldsymbol{\theta}^2(\boldsymbol{\theta}\cdot\mathbf{u})}{\theta_m^3 a} - F\frac{(\boldsymbol{\theta}\cdot\mathbf{u})\mathbf{u}^2}{\theta_m a^3} \tag{9}$$

where B, C, D, E and F are the aberration coefficients corresponding to the spherical aberration, astigmatism, curvature of field, distortion and coma; and $2\theta_m$ is the maximum field of view.

For the purpose of automated design the corrector plate is represented as an $N_1 \times N_2$ array with the resolution

$$\begin{aligned} \delta_1 &= 2a/N_1, \\ \delta_2 &= 2a/N_2, \end{aligned} \tag{10}$$

and m_0 binary digits being used to encode one point of this discrete stencil. Accordingly, the eiconal transmission of the corrector plate has $M = 2^{m_0}$ gradations. Argument sampling and level quantization give rise to the specific unremovable effects that control the limiting characteristics of the corrector plate.

(iii) Because the actual wave aberration of the corrector plate, $\hat{B}_c(\mathbf{u}, \boldsymbol{\theta})$ differs from that required by Eq. (8), and the lens-corrector system will have the residual aberration

$$B(\mathbf{u}, \boldsymbol{\theta}) = \hat{B}_c(\mathbf{u}, \boldsymbol{\theta}) - B_c(\mathbf{u}, \boldsymbol{\theta}). \tag{11}$$

The point spread function (PSF) of this system, therefore, will be somewhat different from the PSF of the perfect aberration-free system. The intensity I at the centre $\mathbf{x}_0$ of the blur spot is the fraction

$$S = I/I_0 \tag{12}$$

of the intensity of the perfect system [6]

$$I_0 = |E_0|^2 \frac{a^4}{\lambda^2 f_0}, \tag{13}$$

whereas the size Δ as a function of the level $0 < \theta < 1$ becomes larger than the blur of the perfect system

$$\Delta_0 = 2\xi_0 f_0/ka, \quad k = 2\pi/\lambda. \tag{14}$$

By way of illustration, $\xi_{0.1} = 2.73$ and $\xi_{0.8} = 0.94$.

When residual aberrations are small, the Strehl ratio may be found as [6]

$$S = \exp(-k^2\bar{B}^2) \simeq 1 - k^2\bar{B}^2, \tag{15}$$

where $\bar{B}^2$ is the mean square aberration.

By equating the light flux passing through the blur spots in both systems under consideration we get

$$\Delta = \Delta_0/\sqrt{S}. \tag{16}$$

In the paraxial approximation, the residual aberration is the error of a piecewise-linear approximation and may be estimated [3] by the formula

$$\bar{B}^2 = \frac{\lambda^2}{12M} + \frac{\delta^2}{12}\int_G |\nabla_u b_l(\mathbf{u},\boldsymbol{\theta})|^2 \mathrm{d}^2\mathbf{u}, \tag{17}$$

where $\delta = \max(\delta_1, \delta_2)$.

After straightforward but unwieldy manipulations, for primary aberrations Eqs (9) and (17) yield

$$\bar{B}^2 = \frac{\lambda^2}{12M^2} + \frac{\delta^2}{6a^2}\left\{\frac{B^2}{8} + \frac{\theta^2}{\theta_m^2}\left[\frac{B(C+D)}{3} + \frac{5F^2}{6}\right] + \frac{\theta^4}{\theta_m^4}\left[\frac{D^2+2DC-2C^2}{4} + EF\right] + \frac{\theta^6}{\theta_m^6}\frac{E^2}{6}\right\}. \tag{18}$$

By way of example consider the case of a corrector plate attached to a planoconvex thin lens with radius of curvature R and refractive index n. Denoting $V = 1/n$ and $f_0 = R/(n-1)$ and making use of the formulae for thin lens aberration coefficients we get

$$\bar{B}^2 = \frac{\lambda^2}{12M^2} + \frac{\delta^2}{6}\left[\frac{1}{32(1-V)^4}\frac{a^6}{f_0^2} + \frac{4.5+V}{12(1-V)}\frac{a^4}{f_0^4}\theta_m^2 + \frac{(2+V)^2+1}{16}\frac{a^2}{f_0^2}\theta_m^4\right]. \tag{19}$$

Equations (12)–(18) and, in particular, Eq. (19) relate the parameters of corrector plate discretization (δ and M) and optical system parameters (field of view θ_m, aperture ratio $2a/f_0$, focal length f_0, operating wavelength λ, and n) to the system's performance measures such as angular resolution Δ/f_0 and Strehl definition S. The computational data summarized in Tables 1 and 2 provide an insight into the dependence of the lens-corrector system on the aforementioned parameters and give guidelines on the resolution requirement of the mask generating facility and

Table 1. $\theta_m = 30°$, $2a/f_0 = 1{:}5$, $M = 8$ and $f_0 = 50$ mm

δ (μm)	5	10	15	25	35	50
Δ/f_0	0.92′	0.95′	0.99′	1.17′	1.48′	2.48′
S	0.93	0.88	0.79	0.58	0.36	0.13
Δ (μm)	13.1	13.4	14.2	16.7	21.1	35.8
B	$\lambda/24$	$\lambda/17$	$\lambda/13$	$\lambda/8$	$\lambda/6$	$\lambda/4$

Table 2. $\theta_m = 30°$, $2a/f_0 = 1:5$, $\delta = 10\ \mu m$ and $f_0 = 50$ mm

M	2	4	6	8
Δ/f_0	1.38′	1.04′	0.97′	0.95′
S	0.41	0.75	0.84	0.88
Δ (μm)	19.9	14.9	13.8	13.5
B	$\lambda/7$	$\lambda/12$	$\lambda/15$	$\lambda/17$

on an appropriate selection of the number of binary masks ($M - 1$) used in the photolithographic fabrication of the plane corrector. For example, Table 1 indicates that for an aperture ratio of 1:5, to achieve angular coverage up to 30° at an angular resolution of 1.5 minutes of arc it would be sufficient to make $\delta = 25\ \mu m$ and $M = 8$, which is well within the capabilities of computer optics technology.

A software package developed for image processing and digital holography [8] was used to create a planar compensator of aberrations for the planoconvex lens specified as above ($n = 1.6$).

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